

DESY 01-007  
 January 2001

hep-ph/0102025

# Twist-4 Gluon Recombination Corrections for Deep Inelastic Structure Functions<sup>1</sup>

J. Blümlein<sup>a</sup>, V. Ravindran<sup>a,b</sup>, Jianhong Ruan<sup>c</sup> and Wei Zhu<sup>a,c</sup>

<sup>a</sup>*DESY-Zeuthen*

*Platanenallee 6, D-15735 Zeuthen, Germany*

<sup>b</sup>*Harish-Chandra Research Institute,*

*Chhatnag Road, Jhusi, Allahabad, 211019, India*

<sup>c</sup>*Department of Physics, East China Normal University,  
 Shanghai 200062, PR China*

## Abstract

We calculate twist-4 coefficient functions for the deep inelastic structure function  $F_2(x, Q^2)$  associated to 4-gluon operator matrix elements for general values of the Bjorken variable  $x$  and study the numerical effect on the slope  $\partial F_2(x, Q^2)/\partial \log Q^2$ . It is shown that these contributions diminish the strongly rising twist-2 terms towards small values of  $x$ .

PACS numbers : 12.38.Aw, Bx, Qk; 13.60.-r

---

<sup>1</sup>Work supported in part by EU contract FMRX-CT98-0194 (DG 12 - MIHT) and the National Natural Science Foundation of China (No. 10075020)

# 1 Introduction

The strong rise of deeply inelastic structure functions  $F_2(x, Q^2)$  and  $F_L(x, Q^2)$  for small values of the Bjorken scaling variable  $x = Q^2/2p.q$  is a consequence of both the structure of the twist-2 ( $\tau = 2$ ) evolution kernels at leading order and the behavior of the non-perturbative flavor singlet input distributions at scales  $Q_0^2 \sim 1 \text{ GeV}^2$ . Potentially this behavior violates unitarity which has to be guaranteed by higher order QCD corrections. As well-known, already the next-to-leading order corrections diminish the rising rate, see e.g. [1]<sup>2</sup>, however, they do not lead to a saturation of  $F_{2,L}(x, Q^2)$  as  $x \rightarrow 0$ .

The large growth of  $F_2(x, Q^2)$  in the double-logarithmic approximation with idealized  $\delta(1-x)$ -like non-perturbative input distributions led Gribov, Levin and Ryskin [3] to propose a non-linear integro-differential equation for the gluon density, which obeyed saturation as  $x \rightarrow 0$ . In this scenario it is assumed that the non-perturbative  $n$ -particle gluon distribution is the  $n$ -th power of the single-particle gluon density. The evolution kernels are derived using the double-logarithmic approximation and an idealized factorization between the non-perturbative input densities and the evolution kernels is assumed. The negative sign of the non-linear gluon-recombination term is introduced referring to the Abramowskii-Gribov-Kancheli cutting rules [4] from Regge theory. The resummation equation can be expanded into an infinite series of so-called fan-diagrams of  $O(1/(Q^2)^k)$ . This equation is of the Glauber-type [5] and not derived as a resummation of certain dominant classes of contributions emerging in the light cone expansion, although the term *twist* is used by some authors to denote the different  $1/(Q^2)^k$  contributions.<sup>3</sup>

Later Mueller and Qiu [6] studied this recombination process too and derived a value for the recombination strength being yet unspecified in Ref. [3]. Durand [7] tried to reverse the mechanism leading to the leading order evolution equations, which, however, turned out to be not possible at this level.<sup>4</sup> A first numerical illustration of the contributions due the resummation in [3] on the gluon density was pursued in Ref. [9]. More detailed investigations, partly leaving the double-log level, were carried out in Refs. [10–12]. The non-linear resummation equation was solved in three different ways in [10] by showing existence and uniqueness of the solution of this equation. An exact value of the resummed gluon density in the saturation limit was derived analytically as well as the solution in the quasi-classic limit was studied leading to a derivation of a critical saturation line as a function of  $x$  and  $Q^2$ .

In a forthcoming investigation of the resummation [3] by Bartels [13], see also [14], it was shown that even using the double-logarithmic approximation other than the fan-diagram contributions were present in the same order though suppressed by a power the number of colors,  $N_c$ . Since the resummation is non-linear these perturbing terms being present in all orders spoil the resummation already in this approximation and quantitative predictions on their effect are hard to make.

All these investigations have to be viewed in the general context of perturbative QCD. Here the question arises to which end the assumptions made can be validated in a rigorous approach. As well-known, the double-logarithmic approximation often yields much larger results than the complete calculation, cf. [15]. In a non-linear iteration quantitative effects due to this difference can be very large. To separate the non-perturbative input distributions and the perturbative evolution kernels and coefficient functions factorization has to be shown for the respective orders in  $1/(Q^2)^k$ . In general a non-universal structure emerges both at the color and Lorentz-level comparing different orders in  $1/Q^2$  and the possibility to resum in general appears to be unlikely.

---

<sup>2</sup>Recent numerical estimates for NNL order [2] continue this trend.

<sup>3</sup>More appropriately one should call these terms power-corrections.

<sup>4</sup>A similar mechanism is, however, applicable in the case of 2-nucleon reactions in a nucleus, [8].

Therefore, in a first step, the simpler question is asked for the structure of the 4 gluon → 2 quark coefficient functions, which relates the 4-gluon operator matrix elements to the deep inelastic structure functions, cf. also Ref. [16]. Here a central question is that for the sign of these coefficient functions as a function of  $x$ , which in a sense was conjectured to be negative in the foregoing literature in the small  $x$  limit. It is clear that one neither can rely on the double-logarithmic approximation nor a small- $x$  approximation in the sense of ‘dominant poles’, as well-known, see Refs. [17, 15, 18], due to the interplay of small  $x$  and large  $x$  contributions via the Mellin-type integrals between the coefficient functions and the respective operator matrix elements. Instead one has to perform a calculation for the whole range of  $x$ .

In this note we present numerical results on the effect of main contributions to the twist-4 coefficient function driven by a two-particle gluon distribution  $G_2$  for the slope  $\partial F_2(x, Q^2)/\partial \log Q^2$ . This quantity is of interest since one may generally expect that higher twist effects are more easily detectable in this distribution than in the integrated structure function  $F_2(x, Q^2)$ . After discussing the structure of the non-perturbative distribution  $G_2$  in section 2 the coefficient function is derived (section 3). Numerical results are presented in section 4 and section 5 contains the conclusions.

## 2 4–Gluon Operator Matrix Elements

The non-perturbative input distributions for a 4-gluon state being linked to a quark box contains four color, four vector indices, and three scaling variables in general. A priori nothing is known about this quantity except the weight factor  $Q_0^2/Q^2$  relative to the twist 2 contributions defined by the non-perturbative parameter  $Q_0^2 = 1/R^2$ . Here  $R$  denotes a twist-4 screening length. In analogy to the single particle gluon density one might define the  $x$ -dependent part for a multi-gluon density depending on  $n$  momentum fractions

$$G_{(n)}(x_1, \dots, x_n) = A_n x_1^{\alpha_1} \dots x_n^{\alpha_n} (1 - x_1 - \dots - x_n)^{\beta} \quad (1)$$

similar to Ref. [19]. One may further assume  $\alpha \equiv \alpha_i, \forall i$  for simplicity. If one attempts to relate the four-gluon distribution

$$\hat{G}_2(x_1, x_2; x'_1, x'_2) = G_2(x_1, x_2; x'_1, x'_2) \delta(x_1 + x_2 - x'_1 - x'_2) \quad (2)$$

to a product of two single-gluon distributions  $G_1(x_i)$  it becomes clear that this is only possible by discarding one of the  $x_i$ 's or relating it to the other two. In a first attempt one still might want to work in such an approximation and even assume that  $G_2$  is found as the simple product of two single gluon densities setting the unknown non-perturbative correlator  $\chi(x_1, x_2, x_3)$  to 1 to obtain first numerical results.

It is evident that on the level of twist-4 the general description requires the introduction of several 4-particle gluon distributions, according to the different color and Lorentz structures, which are not related. This further complicates the determination of these quantities from experimental data. The permutation behavior of the arguments of  $G_2$  being linked to a certain piece of the coefficient function depends on the transformation properties of the latter concerning both color and the momentum fractions  $x_i$ . Given the fact that the correlators  $\chi$  are yet unknown, it is not even clear a priori, whether  $G_2(x_1, x_2, x_3)$  is positive everywhere. With all these reservations in mind we are going to follow a *simplified* approach in the present paper. We will assume  $x_1 = x'_1 = x_2 = x'_2$  and  $\chi \equiv 1$ . This leads to

$$G_2(x_1) = \frac{Q_0^2}{Q^2} G_1^2(x_1) \quad (3)$$

which guarantees the positivity of  $G_2$ . Since the ansatz (3) is not of general validity one has to restrict the investigation using special color and Lorentz contractions in the factorization of the coefficient function and the hadronic matrix elements below. Following [6] we use for the representation of the two-particle (momentum) gluon density

$$\mathbf{G}_2(x_1) = \frac{9}{8} Q_0^2 x_1^2 G_1^2(x_1) . \quad (4)$$

### 3 Coefficient Function

The twist-4 coefficient function describing the 4-gluon  $\rightarrow$  2-quark amplitude is calculated to  $dp_\perp^2/p_\perp^4$  accuracy using time-ordered perturbation theory [20]. A first investigation of the problem has been performed in [21, 22]. The contributing terms are illustrated in Figure 1 (direct terms) and Figure 2 (interference terms). Here the grey ovals symbolize the set of diagrams being shown in Figure 3. The transverse momentum  $p_\perp$  occurs in the internal loop.

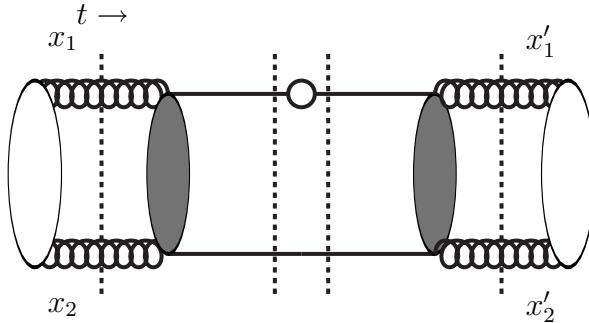


Figure 1: Direct diagrams contributing to (5). The grey oval symbolizes the set of diagrams in Figure 3. Orthogonal dashed lines stand for the time ordering. The separated white ovals symbolize the two parts of the non-perturbative 4-gluon density.  $x_1, x_2, x'_1$  and  $x'_2$  are the longitudinal momentum fractions, with  $x_1 + x_2 - x'_1 - x'_2 = 0$ . The circle stands for the forward subprocess  $\gamma^* + q \rightarrow \gamma^* + q$  through which the virtual photon couples to the amplitude.

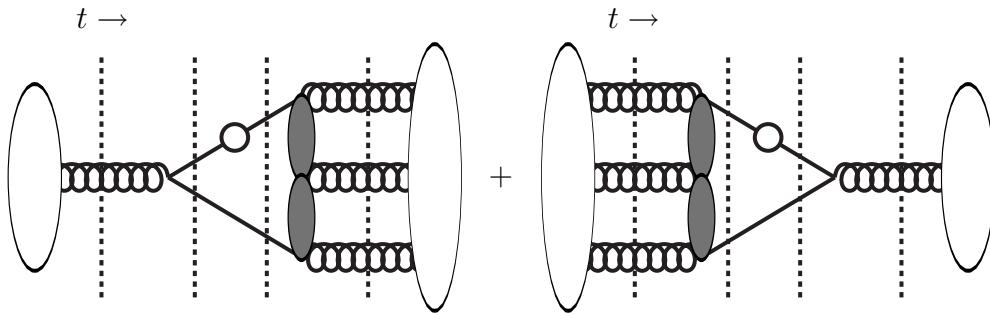


Figure 2: Interference diagrams associated to the process in Figure 1.

The white ovals in Figures 1,2 stand for the respective parts of the non-perturbative distribution  $\mathbf{G}_2$ , which are collinearly factorized. In the calculation we assumed  $x_1 = x'_1$  and  $x_2 = x'_2$ , which is the natural choice, if one aims on using the ansatz  $\mathbf{G}_2(x_1, x_2) = x_1 G_1(x_1) x_2 G_1(x_2)$ . Finally  $x_1$  and  $x_2$  were, furthermore, set equal, in a first approximation, according to the choice Eq. (4) for  $\mathbf{G}_2$ .

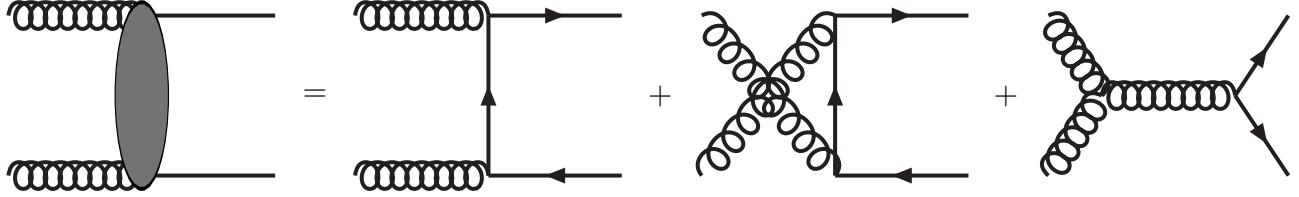


Figure 3: Diagrams symbolized by grey ovals in Figure 1 and 2.

We have independently recalculated the coefficient functions for these processes. For the  $2 \rightarrow 2$  coefficient function one obtains :

$$C_{G \rightarrow q}^{4 \rightarrow 2; 2 \rightarrow 2}(x_1, x_1; x'_1, x'_1; x) = C_{G \rightarrow q}^{4 \rightarrow 2; 2 \rightarrow 2}(x_1, x) = \frac{1}{96} \frac{(2x_1 - x)^2}{x_1^5} [14x^2 - 3xx_1 + 18x_1^2]. \quad (5)$$

Under the above assumptions the 3rd diagram in Figure 3 does not contribute. Eq. (5) can be given a simple form introducing the variable  $z = x/x_1$ . Note that the Mellin poles in this variable are situated at  $N = 0, -1, \dots, -4$  and are of similar strength. Non of these poles is therefore dominant. For the direct process the kinematic range of the variable  $x_1$  is  $x_1 \in [x/2, 1/2]$ .

The interference contributions, Figure 2, consist out of the coefficient function for the  $1 \rightarrow 3$  and  $3 \rightarrow 1$  process and the factorized non-perturbative distributions. One may show, cf. [21], that these coefficient functions are related to that of the  $2 \rightarrow 2$  process by a sign-change in the *whole*  $x$  range.

$$C_{G \rightarrow q}^{4 \rightarrow 2; 2 \rightarrow 2}(x_1, x_1; x'_1, x'_1; x) = -C_{G \rightarrow q}^{4 \rightarrow 2; 1 \rightarrow 3}(x_1, x_1; x'_1, x'_1; x) = -C_{G \rightarrow q}^{4 \rightarrow 2; 3 \rightarrow 1}(x_1, x_1; x'_1, x'_1; x). \quad (6)$$

However, the kinematic range of  $x_1$  is now  $x_1 \in [x, 1/2]$ . Arguments were given in Ref. [23] that one may choose the same non-perturbative input for the direct and interference terms. This hypothesis is being adopted here. For the slope  $\partial F_2(x, Q^2)/\partial \log Q^2$  one finally obtains :

$$\begin{aligned} \frac{\partial F_2(x, Q^2)}{\partial \log(Q^2)} &= \frac{\partial F_2^{\tau=2}(x, Q^2)}{\partial \log(Q^2)} + \left(\frac{\alpha_s}{2\pi}\right)^2 \frac{1}{Q^2} \int_{x/2}^{1/2} dx_1 \left(\frac{x}{x_1}\right) C_{G \rightarrow q}^{4 \rightarrow 2; 2 \rightarrow 2}(x_1, x) \mathbf{G}_2(x_1) \\ &\quad - 2 \left(\frac{\alpha_s}{2\pi}\right)^2 \frac{1}{Q^2} \int_x^{1/2} dx_1 \left(\frac{x}{x_1}\right) C_{G \rightarrow q}^{4 \rightarrow 2; 2 \rightarrow 2}(x_1, x) \mathbf{G}_2(x_1), \end{aligned} \quad (7)$$

where

$$\frac{\partial F_2^{\tau=2}(x, Q^2)}{\partial \log(Q^2)} = \left(\frac{\alpha_s}{2\pi}\right) x \left\{ \sum_q e_q^2 [P_{qq} \otimes (q + \bar{q})](x) + \left[ \sum_q e_q^2 \right] [P_{qG} \otimes G_1](x) \right\}, \quad (8)$$

and  $\otimes$  denotes the Mellin convolution.

## 4 Numerical Results

Numerical values for the slope  $\partial F_2(x, Q^2)/\partial \log Q^2$  are given in Figure 4. Here we compare the results in leading order QCD for the twist-2 contribution to those obtained including the twist-4 term Eq. (7). For the twist-2 contribution and the non-perturbative distribution  $\mathbf{G}_2(z, Q^2)$  we

refer to the parameterization [24] and Eq. (4), respectively. Figure 4a and b depict the results for  $Q^2 = 5 \text{ GeV}^2$  and  $10 \text{ GeV}^2$ , choosing different screening lengths  $R$ . Under the above assumptions the twist-4 correction is negative in the small- $x$  range, i.e. the screening term in Eq. (7) wins against the anti-screening term. The latter one leads to a very small positive contribution only at large values of  $x$ . The effect shrinks with rising values of  $Q^2$ , and becomes visible only for  $x$  values below  $x \sim 10^{-4}$  at perturbative scales of  $Q^2 \gtrsim 4 \text{ GeV}^2$ . Therefore this effect cannot be seen yet in the kinematic range of HERA. Although one might find arguments for the size of the screening parameter  $R$ , it is a non-perturbative quantity for which a general prediction within QCD is still missing. We gave illustrations for  $R^2 = 5$  and  $2 \text{ GeV}^2$ . In the latter case a clear effect is visible for  $x \leq 2 \cdot 10^{-6}$  and  $Q^2 \sim 5 \text{ GeV}^2$ .

The coefficient function Eqs. (5,6) applies to the whole  $z$ -range with  $z = x/x_1$ . One might try to find an effective small- $x$  approximation. The function is a polynomial of degree 4 in the variable  $z$ . Approximating it successively up to its  $z^0, z, z^2 \dots$  -contribution one may study the effect of the individual terms. Figure 5 shows, that a sufficient representation is only given taking all contributions into account since all approximants diverge for  $x \rightarrow 0$ , i. e. an effective small- $x$  approximation to this function does not exist.

## 5 Conclusions

A study has been performed to the slope of  $F_2$  due to twist-4 coefficient functions for the process  $4G \rightarrow 2q$ . The present calculation was performed in time-ordered perturbation theory and limited to  $dp_\perp^2/p_\perp^4$  accuracy of the respective Feynman diagrams. Numerical results were choosing a special ansatz for the non-perturbative two-particle gluon density  $\mathbf{G}_2$ . The new contributions contain anti-screening and screening terms, the latter of which dominate for smaller values of  $x$  and diminish the growth of the slope due to the twist-2 contributions. The numerical results show that this effect is of significant size only below  $x \sim 2 \cdot 10^{-6}$  for  $Q^2 \sim 5 \text{ GeV}^2$ . This region is yet beyond the kinematic range which can be probed at the  $ep$  collider HERA but may be investigated at future lepton-hadron facilities operating at larger cms energies.

**Acknowledgment:** For discussions we would like to thank W.L. van Neerven, A. Vogt and A.H. Mueller. J.B. is thankful to R. Devenish for financial support during the 2000 Oxford Small  $x$  Meeting, where first results on this work were presented.

## References

- [1] Wu-Ki Tung, Nucl. Phys. **B315** (1989) 378.
- [2] W.L. van Neerven and A. Vogt, Phys. Lett. **B490** (2000) 111.
- [3] L. Gribov, E. Levin, and M. Ryskin, Nucl. Phys. **B188** (1981) 555.
- [4] V. Abramovsky, V. Gribov, and O.Kancheli, Sov. J. Nucl. Phys. **18** (1974) 308;  
J. Bartels and M. Ryskin, Z. Phys. **C76** (1997) 241.
- [5] R. Glauber in: *Lectures in Theoretical Physics*, eds. W.E. Britten and G. Dunham (Interscience, New York, 1959);  
A.H. Mueller, Nucl. Phys. **B335** (1990) 115.
- [6] A.H. Mueller and J. Qiu, Nucl. Phys. **B268** (1986) 427.

- [7] L. Durand, MAD/TH/208 (1984), in: Proc. Snowmass Summer Study 1984: 0256.
- [8] F. Close, R. Roberts, and J. Qiu, Phys. Rev. **D40** (1989) 2820.
- [9] J. Kwiecinski, Z. Phys. **C29** (1985) 147.
- [10] J. Bartels, J. Blümlein and G. Schuler, Z. Phys. **C50** (1991) 91.
- [11] C. Collins and J. Kwiecinski, Nucl. Phys. **B335** (1990) 89.
- [12] M. Altmann, M. Glück, and E. Reya, Phys. Lett. **B285** (1992) 359.
- [13] J. Bartels, Phys. Lett. **B298** (1993) 204.
- [14] E. Levin, M. Ryskin and A. Shuvaev, Nucl. Phys. **B387** (1992) 589.
- [15] J. Blümlein and A. Vogt, Phys. Rev. **D58** (1998) 014020.
- [16] J. Bartels and C. Bontus, Phys. Rev. **D61** (2000) 034009;  
J. Bartels, C. Bontus, and H. Spiesberger, [hep-ph/9908411](#).
- [17] J. Blümlein, talk, Durham HERA Workshop, March 1993 and J. Phys. **G19** (1993) 1695;  
W.L. van Neerven, talk, DESY Theory Workshop, September 1993.
- [18] J. Blümlein and W.L. van Neerven, Phys. Lett. **B450** (1999) 412;  
J. Blümlein, S. Riemersma, and A. Vogt, [hep-ph/9608470](#), Nucl. Phys. **B** (Proc. Suppl.), **51C** (1996) 30;  
J. Blümlein and A. Vogt, Phys. Rev. **D57** (1998) R1; Phys. Lett. **B370** (1996) 149; Acta Phys. Pol. **B27** (1996) 1309;  
R.K. Ellis, F. Hautmann and B. Webber, Phys. Lett. **B348** (1995) 582;  
J. Blümlein, V. Ravindran, W.L. van Neerven, and A. Vogt [hep-ph/9806368](#), in: Proc. of the Int. Conference DIS98, Brussels, eds. Gh. Coremans and R. Roosen, (Singapore, World Scientific, 1998), pp. 211;  
J. Blümlein, [hep-ph/9909449](#), in Proc. of the Workshop New Trends in HERA Physics 1999, eds. G. Grindhammer, B. Kniehl and G. Kramer, (Springer, Berlin, 2000) pp. 42.
- [19] J. Kuti and V. Weisskopf, Phys. Rev. **D4** (1971) 3418.
- [20] S. Weinberg, Phys. Rev. **150** (1966) 1313.
- [21] Wei Zhu, Nucl. Phys. **B551** (1999) 245.
- [22] Wei Zhu and Jianhong Ruan, Nucl. Phys. **B559** (1999) 378.
- [23] R.L. Jaffe, Nucl. Phys. **B229** (1983) 205.
- [24] M. Glück, E. Reya, and A. Vogt, Eur. Phys. J. **C5** (1998) 461.

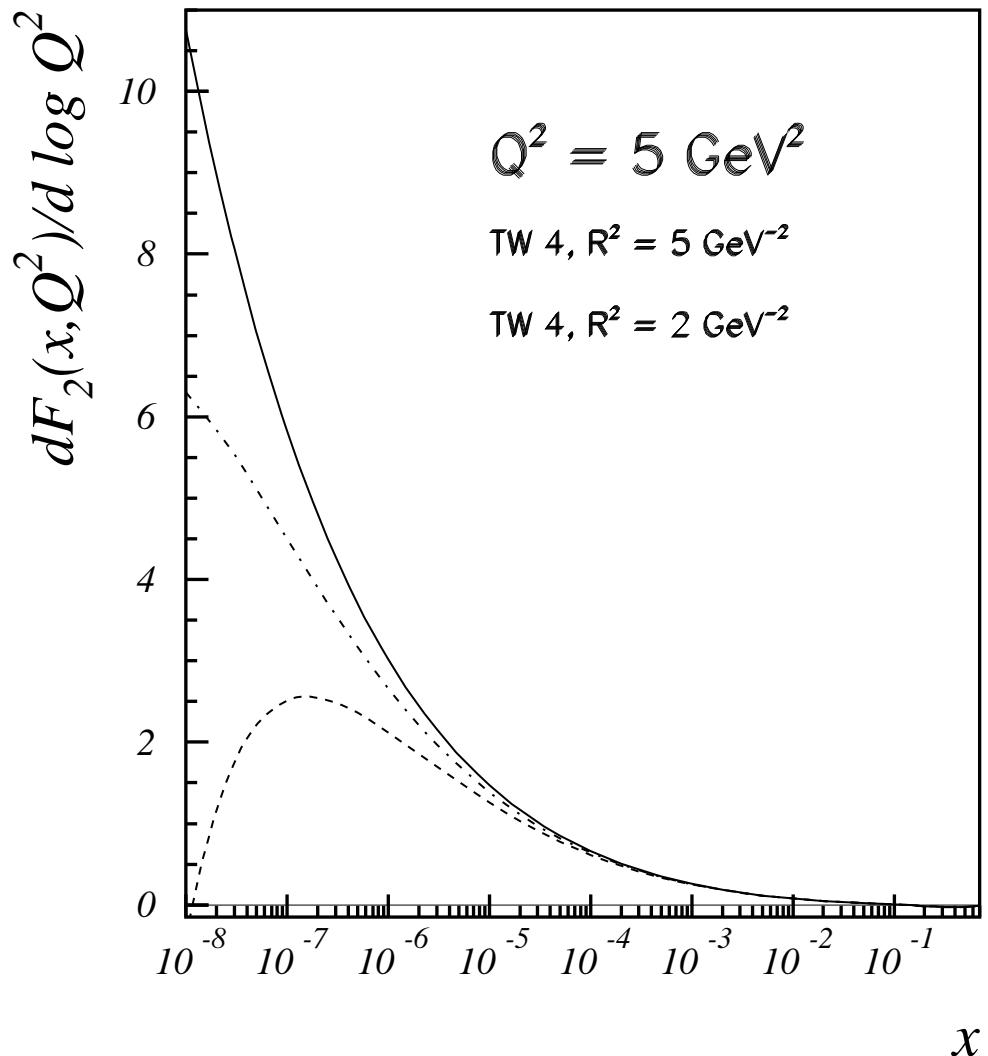


Figure 4a: The slope  $dF_2(x, Q^2)/d \log Q^2$  at  $Q^2 = 5 \text{ GeV}^2$ . Full line: leading order twist-2 contributions (parameterization Ref. [24]). Dash-dotted line: Eq. (7) with twist-4 mass scale  $R^2 = 5 \text{ GeV}^{-2}$ , and dashed line:  $R^2 = 2 \text{ GeV}^{-2}$ .

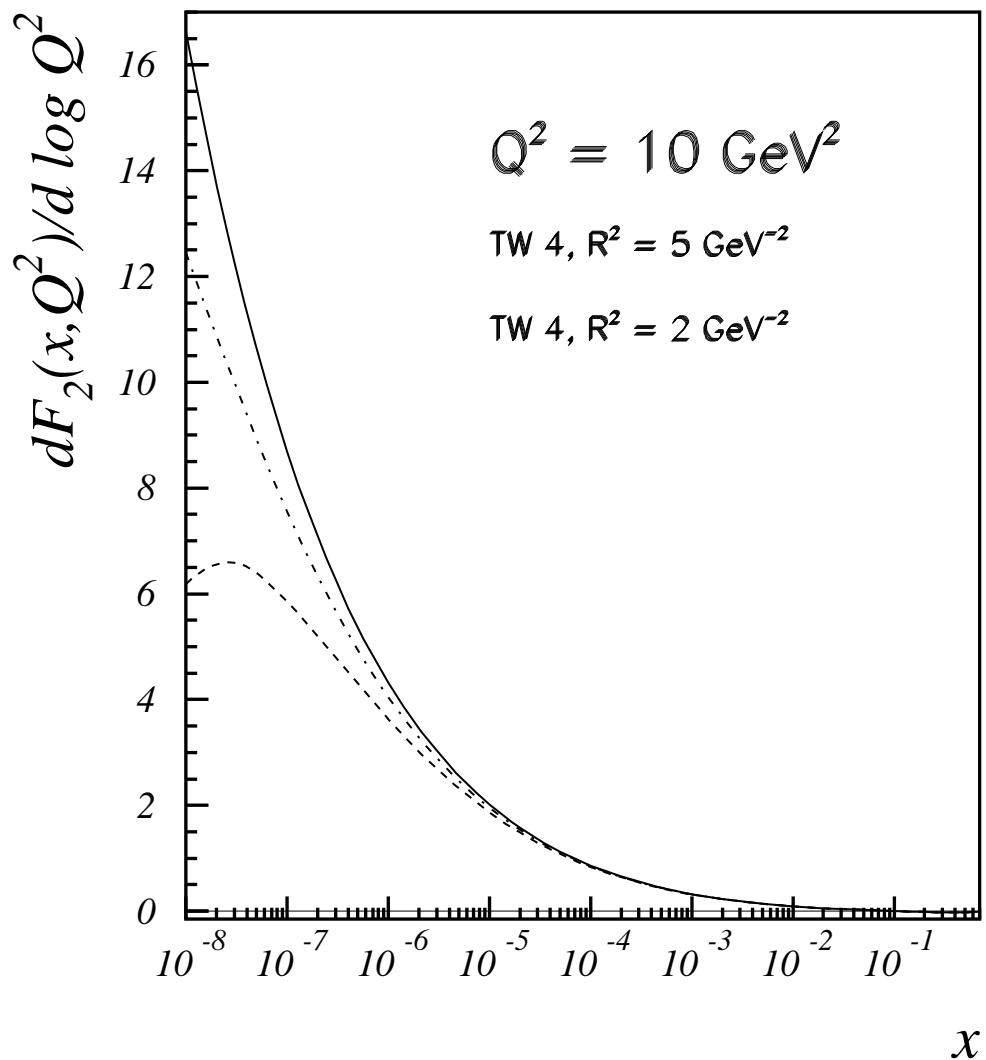


Figure 4b: Same as figure 1a for  $Q^2 = 10 \text{ GeV}^2$ .

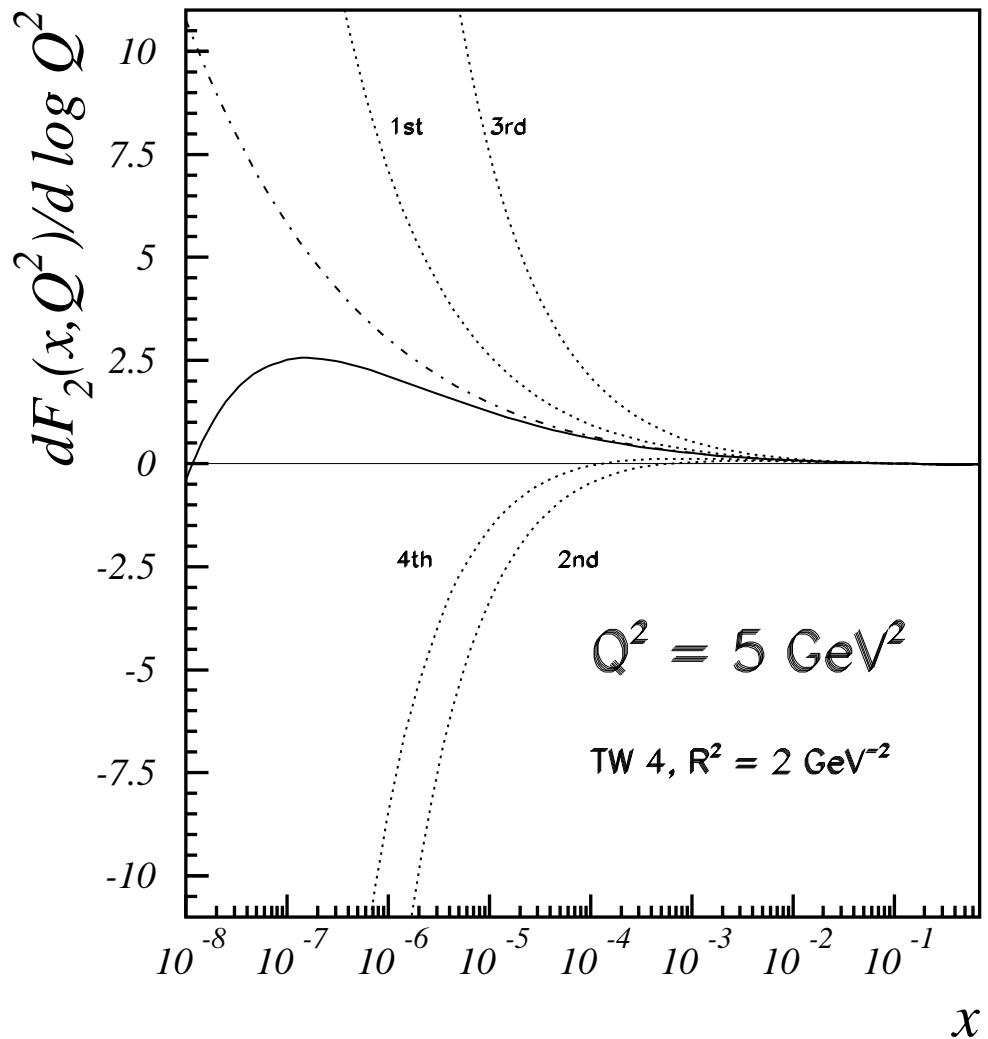


Figure 5: Comparison of the slope  $dF_2(x, Q^2)/d \log Q^2$  at  $Q^2 = 5 \text{ GeV}^2$  and twist-4 mass scale  $R^2 = 2 \text{ GeV}^{-2}$ , Eq. (7) (full line) with the corresponding results obtained approximating the coefficient function Eq. (5) by the sequence of contributing powers. 1st:  $z^0$ , 2nd:  $z$  etc. (dotted lines). Dash-dotted line: twist-2 contribution.